

# Estimating fundamental matrix from uncalibrated images

Jiangming Kan, Chuandong Zhan, Shuo Feng, Wenbin Li \*

School of Technology, Beijing Forestry University, Beijing, 100083 China

Received 1 June 2014, www.cmnt.lv

---

## Abstract

Estimation of the fundamental matrix plays a significant role in the field of computer vision. Two different approaches are presented to estimate the fundamental matrix from uncalibrated images: one is an improved iterative approach; the other an improved robust estimation. The improved iterative approach, utilizing the least-squares technique, makes use of several point matches to compute the initial fundamental matrix and weights and determines the computation loop by concerning the Euclidean distance between matched points and epipolar lines. The improved robust estimation extends the original RANSAC approach by removing outliers from the points set every five inner loops after being evaluated with corresponded scores to get the optimal points set and then estimating the fundamental matrix through the orthogonal least-square algorithm at each iteration. Experimental results that the improved iterative performs better when both the variance of Gaussian noises and the percentage of outliers are small. Results reveal that the proposed technique of removing outliers works successfully and fine, especially with a high level of outliers; and it is superior to the original RANSAC in terms of means and standard deviation on real images.

*Keywords:* epipolar geometry, fundamental matrix, iterative computation, robust estimation

---

## 1 Introduction

The fundamental matrix encapsulates the constraint obeyed by image point correspondences if they are to be images of the same 3D point arising from the co-planarity of the camera centers of the two views, the images points and the space point known as epipolar geometry [1]. Epipolar geometry can be obtained from the image correspondences when the camera parameters and the motion between them are unknown. Besides, Epipolar geometry is independent from the structure of the scene and it relates only to the internal parameters and relative pose of the two camera systems. The fundamental matrix can be estimated from the point correspondences of the two images without knowing the internal information of the cameras in advance.

Recent research on estimating the fundamental matrix can be classified into linear methods, iterative methods and robust methods. The linear methods include the seven-point algorithm, eight-point algorithm, least-square technique, orthogonal least-square technique and analytic method with the rank-2 constraint [2]. These leaner methods are time-saving but sensitive to noises and a problem with the linear criterion is that the quantity minimized is not physically meaningful. The iterative methods are classified into two groups: those minimize the distance between points and epipolar lines and those are based on the gradient [3]. The iterative methods have improved the accuracy compared with the linear methods, but they consume more time and cannot deal with outliers and have to solve the initial fundamental matrix and weights as well as conditions for looping termination. The

robust methods can cope with outliers, bad locations and false matching as well. Generally used robust methods include M-Estimators, Least-Median-Squares (LMedS), Random Sampling Consensus (RANSAC), MLESAC and MAPSAC [3].

Thus, this paper presents two different approaches to estimate the fundamental matrix: one is based on iterative computation; the other is robust estimation. The iterative approach, utilizing the least-square technique, extracts a set of point matches to compute the initial fundamental matrix and weights and determines the computation loop by concerning the Euclidean distance between matched points and epipolar lines. The robust estimation that is an improved RANSAC approach, adopts a stricter way to remove outliers from the initial matched points in order to achieve the optimal points set. Experimental results on both real image and synthetic data reveal that our proposed two methods are superior to corresponded original ones in terms of accuracy and robustness against Gaussian noises.

## 2 Epipolar geometry and the fundamental matrix

The epipolar geometry between two views is essentially the geometry of the inter-section of the image planes with the pencil of planes having the baseline as axis [6]. The epipolar geometry exists between the two camera systems focusing on the same scene. Consider that  $C$  and  $C'$  are the optical centres of two cameras shown in Figure 1. Given a point  $\mathbf{m}$  in the first image, its corresponding point  $\mathbf{m}'$  on the second image lies on a line called the epipolar line of  $\mathbf{m}$ , denoted by  $\mathbf{l}'$ . This is known as the epipolar

---

\* Corresponding author's e-mail: leewb@bjfu.edu.cn

constraint [1]. In order to map  $\mathbf{m}$  with  $\mathbf{m}'$ , the following equations must be satisfied in Equation (1).

$$\mathbf{m}^T \mathbf{F} \mathbf{m} = 0 \tag{1}$$

The epipolar geometry is contained in the fundamental matrix  $\mathbf{F}$  and  $\mathbf{F}$  is a  $3 \times 3$  matrix of rank 2 (i.e.  $\det(\mathbf{F}) = 0$ ).

In the last few years, several methods have been proposed and all the methods of estimating the fundamental matrix are based on solving a homogeneous system of equations: Equation (1). The equations can be rewritten in the following way as Equation (2).

$$\mathbf{U}_n \mathbf{f} = 0, \tag{2}$$

where

$$\mathbf{f} = (\mathbf{F}_{11}, \mathbf{F}_{21}, \mathbf{F}_{31}, \mathbf{F}_{12}, \mathbf{F}_{22}, \mathbf{F}_{32}, \mathbf{F}_{13}, \mathbf{F}_{23}, \mathbf{F}_{33})^T \tag{3}$$

$$\mathbf{U}_n = \begin{bmatrix} u_1 u'_1 & u_1 v'_1 & u_1 & v_1 u'_1 & v_1 v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2 u'_2 & u_2 v'_2 & u_2 & v_2 u'_2 & v_2 v'_2 & v_2 & u'_2 & v'_2 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ u_n u'_n & u_n v'_n & u_n & v_n u'_n & v_n v'_n & v_n & u'_n & v'_n & 1 \end{bmatrix}_{n \times 9} \tag{4}$$

This set of linear homogeneous equations and the rank constraint of the matrix  $\mathbf{F}$  allow us to estimate the epipolar geometry. There are 9 unknown parameters, but because of the scale factor forcing the fundamental matrix to be rank-2, there are only 7 independent parameters which are given by two independent columns.

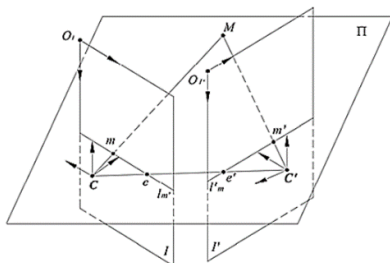


FIGURE 1 Epipolar Geometry

### 3 Related methods

#### 3.1 AN IMPROVED ITERATIVE METHOD

Iterative methods fully take advantages of constraints parameters of the fundamental matrix through well parameterization with the rank-2 constraint, and they can be classified into two groups: the first one is based on minimizing the distances between points and epipolar lines; the second one is the gradient. Luong and Faugeras's weighted linear iterative approach is that the initial weights are all set to be 1 against varying situations and the starting fundamental matrix is given using the eight-point algorithm [4]. Our method carries forward their iterative approach as follows. Firstly, a weighing fundamental matrix  $\mathbf{F}'$  is computed through the least-square algorithm

to calculate the weighted Euclidean distance between matched points and epipolar lines and the standard deviation  $\sigma$  [3]. Secondly, matched points set  $\mathbf{M}$  is divided into two subsets ( $\mathbf{M}_1, \mathbf{M}_2$ ) according to  $\sigma$ . Weighted distances of points in subset  $\mathbf{M}_1$  are greater than  $\sigma$ ;  $\mathbf{M}_2$  consists of points which distances are no more than  $\sigma$ . Thirdly, the mean weights  $\mathbf{w}_0$  are to be computed using subset  $\mathbf{M}_1$  and then estimate the initial iterative  $\mathbf{F}_0$  through the linear least-square technique. Fourthly, iterative calculation of  $\mathbf{F}_i$  is performed using the subset  $\mathbf{M}_2$  and figure out the Euclidean distance  $E_{d,i}$  between  $\mathbf{F}_{i-1}$  and terminate the iteration loop if  $E_i$  is greater than the given threshold value  $e$ . Detailed procedures are shown in Figure 2.

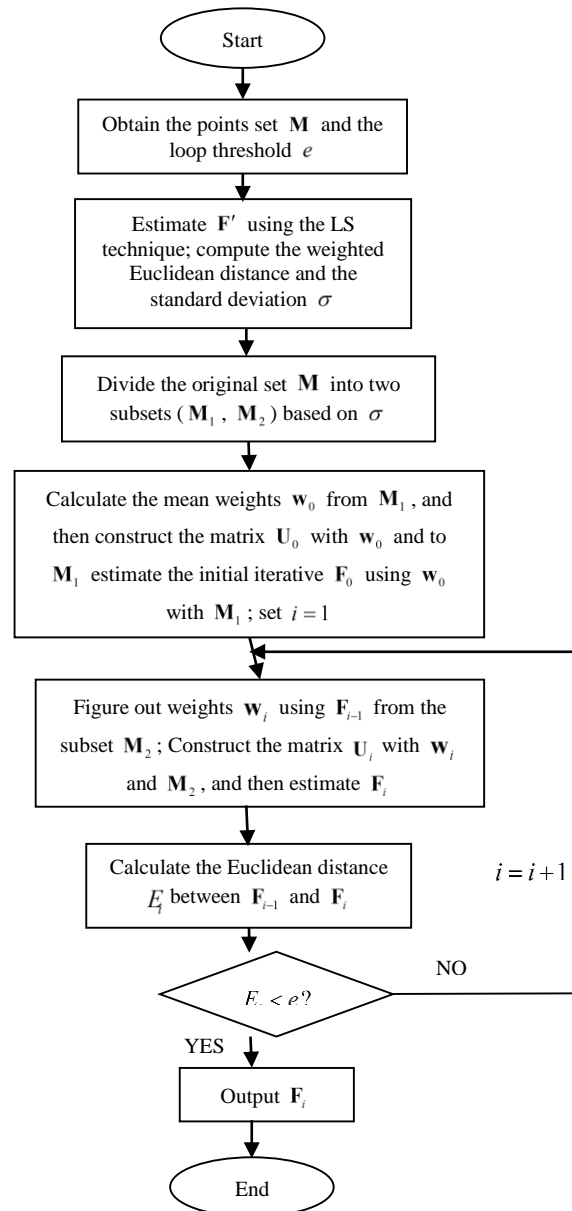


FIGURE 2 Flow scheme of the improved iterative method

3.2 A BETTERED RANSAC APPROACH

The RANSAC technique originally proposed by Fischler and Bolles [8] has been widely used for robust parameter estimation to overcome the outlier problem. The basic idea of using RANSAC for fundamental matrix estimation is as follows: randomly selecting a number of minimal subsets of point correspondences to determine the fundamental matrix for each subset, and then find the best fundamental matrix that is most consistent with the entire set of point correspondences [9]. Detection of outliers is merely performed at a time in every loop during the RANSAC process, meaning that removing outliers only depends on the single detection at each loop.

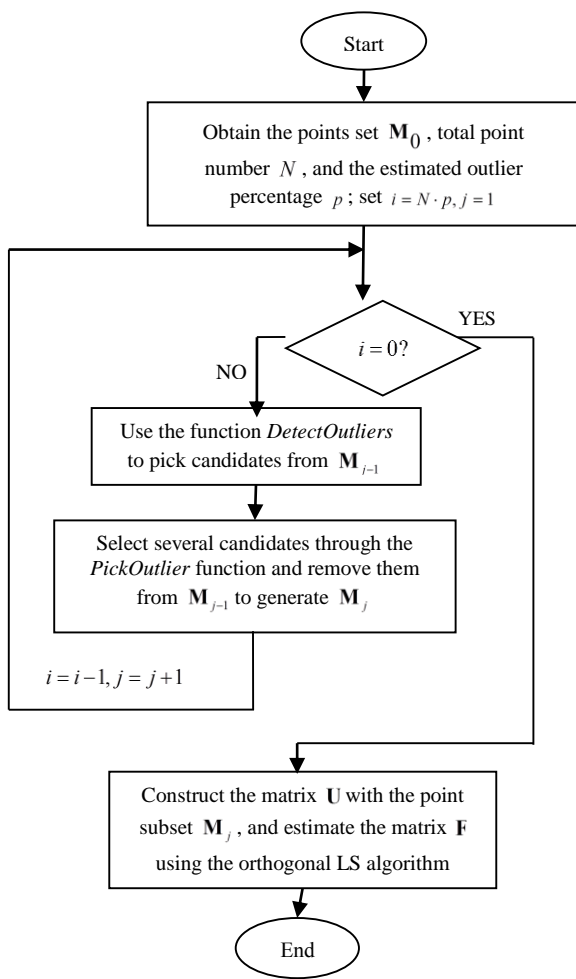


FIGURE 3 Principle flow scheme of the improved RANSAC approach

Our approach extends the original RANSAC technique by removing outliers from the initial points set every five loops after being evaluated to get the optimal points set and then estimates the fundamental matrix through the orthogonal least-square algorithm. The principle flow scheme of our approach is shown in Figure 3 and the detection of outlier candidates can be seen in Figure 4.

Figure 5 illustrates how to pick out exterior points (outliers). Evaluate repentances of candidates in the candidate matrix  $S_{out}$  as their scores in order to pick out outliers to be removed in the point subset  $M_j (j \geq 1)$  of the  $j$ th iteration.

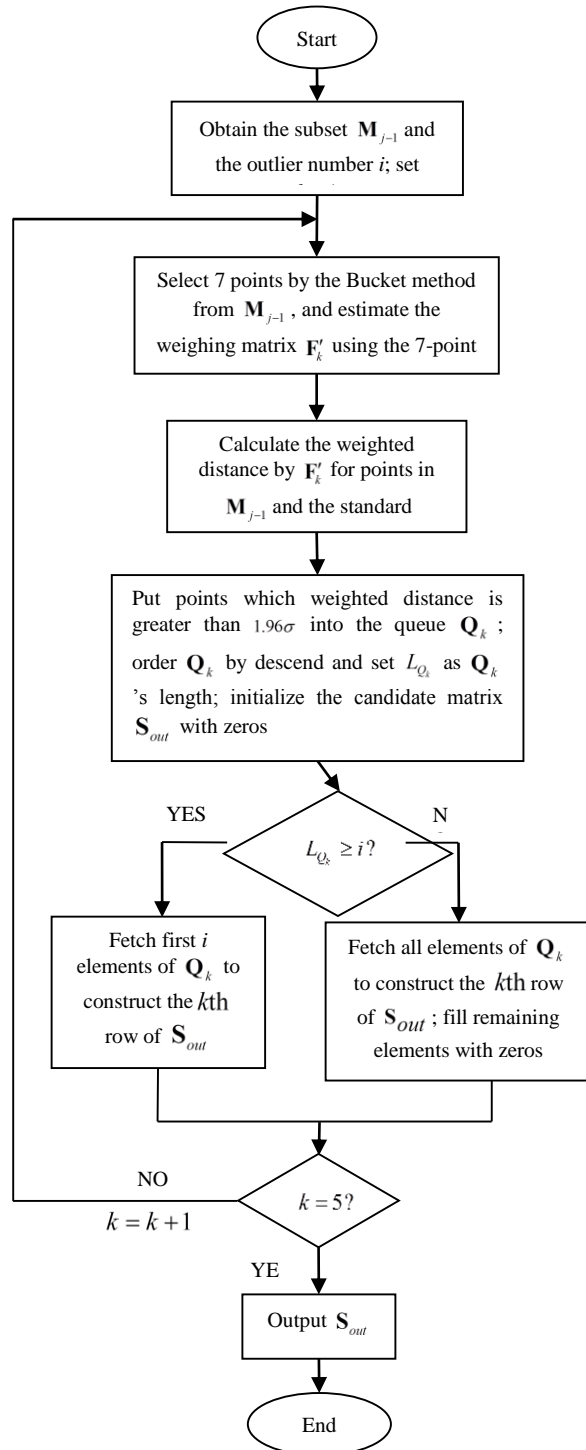


FIGURE 4 Flow scheme of the DetectOutliers function

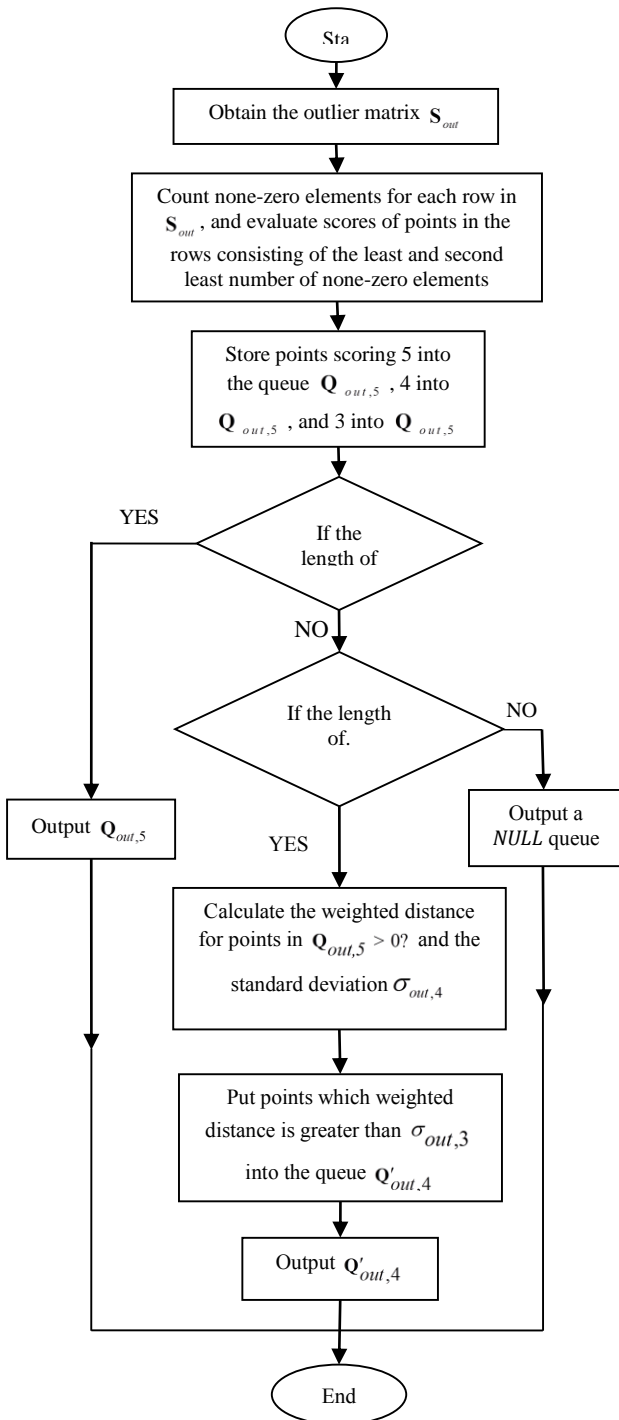


FIGURE 5 Flow scheme of the PickOutlier function

### 4 Experiments

In this paper, both synthetic data and real images are used to perform the experiments of both proposed methods and original ones. The accuracy of each method is considered as the mean and standard deviation of the discrepancy between points and epipolar lines, and the robustness is the accuracy under different ranges of Gaussian noises using

synthetic data. In order to evaluate the time consumed for estimating the fundamental matrix with varied number of points, experiments are repeated for 100 times to get the average values.

#### 4.1 EXPERIMENTS WITH SYNTHETIC DATA

In the experiments with synthetic data, the point matches are randomly generated by space points in 3D space which are visible to two different positions of a synthetic camera [2]. Concerning iterative methods, 100 pairs of matched points are used to estimate the fundamental matrix through both the original and proposed. As for RANSAC approaches, 300 pairs are utilized for estimation of the fundamental matrix. The sensitivity to different ranges of Gaussian noises is also covered in experiments. Different ranges of Gaussian noises are added to the point correspondences, whose means are 0 and variances vary from 0 to 1.

#### 4.2 EXPERIMENTS WITH REAL IMAGES.

Real images are captured by two cameras of an identical scene at different views, are used to estimate the fundamental matrix, and then the point correspondences of the images are detected and matched according to [10-12]. As for iterative methods, 250 pairs of matched points are used to estimate the fundamental matrix through both the original and proposed; for RANSAC approaches, 89 pairs are utilized. Experimental results that are related to accuracy and robustness of algorithms are also collected such as the mean and standard deviation of the discrepancy between points and epipolar lines.

### 5 Results

#### 5.1 ITERATIVE METHOD

##### 5.1.1 Synthetic data

Two fundamental matrices with synthetic data are produced as follows:  $F_{it}$  is computed by the original iterative method;  $F_{pro}$  is estimated using our improved iterative method.

$$F_{it} = \begin{pmatrix} 3.8535e-005 & 1.1564e-004 & 2.3757e-002 \\ -2.4031e-004 & 1.0032e-004 & -1.4350e-002 \\ -1.3386e-001 & 1.1110e-001 & 1.0000e+000 \end{pmatrix}.$$

$$F_{pro} = \begin{pmatrix} 3.8535e-005 & 1.1564e-004 & 2.3757e-002 \\ -2.4031e-004 & 1.0032e-004 & -1.4350e-002 \\ -1.3386e-001 & 1.1110e-001 & 1.0000e+000 \end{pmatrix}.$$

The fundamental matrices above satisfy the constraint of rank-2. In order to evaluate the sensitivity to noises and outliers of the proposed method, the noise-outlier-containing data as shown in Table 1 are added into

experiments and the mean and standard deviations of the discrepancy between points and epipolar lines are computed using both fundamental matrices. In most cases of noises and outliers, the proposed approach performs better than the original in terms of the mean and standard deviation. When both the variance of Gaussian noises and the percentage of outliers go down, performances of the

proposed become even better, which can be inferred from results of the case ( $\sigma = 0.0, 10\%$ ). When the percentage rises with a fixed variance, performances of both approaches decline in an almost identical gradient. When the variance ascends with a constant percentage of outliers, the fundamental matrix estimated by the proposed loses more accuracy that means it is more sensitive to noises.

TABLE 1 Means and standard deviations of the discrepancy between points and epipolar lines using synthetic data

Variance		$\sigma = 0.0$	$\sigma = 0.0$	$\sigma = 0.1$	$\sigma = 0.1$	$\sigma = 0.5$	$\sigma = 0.5$
Outlier Percentage		10%	50%	10%	50%	10%	50%
Original	Mean	51.6283	92.2913	60.3065	86.1131	76.4548	83.8917
	STDEV	47.4254	68.2687	55.6416	65.5661	56.6761	80.7522
Improved	Mean	44.7236	82.2232	66.2452	83.0093	86.5723	80.5916
	STDEV	40.5370	60.9599	57.4684	63.7591	61.4099	64.7965

5.1.2 Real Images

As shown in Figure 6, two images, which are captured by two cameras of an identical scene at different views, are used to estimate the fundamental matrix, and then the point correspondences of the images are detected and matched according to [10-12]. Use all 250 point matches to estimate the fundamental matrix using our proposed method together with the original iteration and the least-square approach. Fundamental matrices are estimated as follows:  $F_{it}$  is generated by the original iterative method;  $F_{pro}$  is figured out through the proposed.

$$F_{it} = \begin{pmatrix} 4.7149e-007 & 1.4063e-006 & -4.0776e-004 \\ 8.0341e-006 & 2.4511e-005 & -6.9571e-003 \\ -1.1526e-003 & -3.5326e-003 & 1.0000e+000 \end{pmatrix}$$

$$F_{pro} = \begin{pmatrix} 5.4044e-007 & 2.0851e-006 & -5.4099e-004 \\ 6.6304e-006 & 2.6149e-005 & -6.8650e-003 \\ -9.7893e-004 & -3.8016e-003 & 1.0000e+000 \end{pmatrix}$$

Figure 7 demonstrates the means between points and the epipolar lines using both methods in every iterative procedure. Though the means of the presented bump in the beginning iterations and are greater than that of the original, from the fifth iteration the means begin reducing and when it comes to 10 iterations both methods perform nearly the same. Moreover, since 13 iterations, the means of the proposed are smaller than that of the original and convergent reaching 17 iterations. This reveals that the proposed needs more iteration so that the consumed time increases as well.



FIGURE 6 Real images used in this experiment

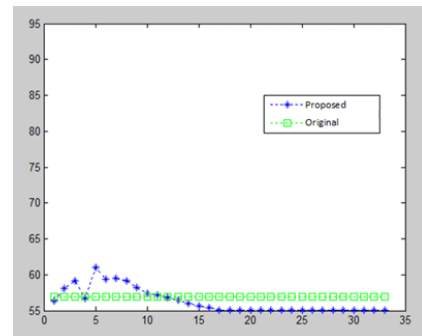


FIGURE 7 Means between points and epipolar lines using both methods

Table 2 shows the means and standard deviations of the discrepancy between points and epipolar lines under both methods as well as the time of estimating the fundamental matrix with ranging number of points. The means of the presented are 55.0693 that are smaller than that of the original 56.8739; the standard deviations are also smaller. It is reasonable to say that the proposed performs better in terms of means and standard deviations and runs more accurate and robust than the original.

TABLE 2 Means and standard deviations of the discrepancy between points and epipolar lines using real images with the time consumed

Algorithm	Original Iterative	Improved
Mean	56.8739	55.0693
STDEV	43.8902	42.0962
Time Consumed (s)	0.0194	0.1572

5.2 RANSAC APPROACH

5.2.1 Synthetic data

Two fundamental matrices with synthetic data are produced as follows:  $F_{RANSAC}$  is computed by the original RANSAC approach;  $F_{pro}$  is estimated using our improved approach.

$$F_{RANSAC} = \begin{pmatrix} -4.3467e-006 & 4.0600e-006 & -9.3295e-004 \\ 2.6873e-005 & -3.2504e-005 & -6.5798e-003 \\ 4.0246e-003 & -3.6775e-003 & 9.9996e-001 \end{pmatrix}$$

$$F_{pro} = \begin{pmatrix} -4.3469e-006 & 4.0602e-006 & -9.3298e-004 \\ 2.6874e-005 & -3.2505e-005 & -6.5801e-003 \\ 4.0248e-003 & -3.6776e-003 & 1.0000e+000 \end{pmatrix}$$

The sensitivity to noises and outliers of the proposed method are evaluated with the noise-outlier-containing data and the mean and standard deviations of the discrepancy between points and epipolar lines are computed using both fundamental matrices above as shown in Table 3. Given the initial points set  $M_0$ , when percentages of outliers are less than 30%, the proposed performs worse; when there are more than 30% outliers, its performances are better. Nevertheless, the means and the standard deviations of the proposed processed with the generated final set  $M_j$  are greater than that of the original. Besides, the means and standard deviations of the original RANSAC approach reduce sharply with the final set  $M_j(j \geq 0)$  that is generated using the improved. This reveals that the subset  $M_j$  consists of much more interior points than  $M_0$  does, inferring that the algorithm of removing outliers used in our approach works fine.

TABLE 3 Means and standard deviations of the discrepancy between points and epipolar lines using synthetic data

Points set	Variance		$\sigma = 0.0$	$\sigma = 0.0$	$\sigma = 0.5$	$\sigma = 0.5$
	Outlier Percentage		10%	20%	30%	40%
$M_0$	Original RANSAC	Mean	26.4562	9.4699	20.8774	24.8397
		STDEV	39.8356	27.1077	31.0247	43.2012
	Improved	Mean	68.3065	17.8038	19.2313	19.4201
		STDEV	95.7710	38.6972	31.5069	40.0162
$M_j(j \geq 0)$	Original RANSAC	Mean	1.9765	0.5157	2.2693	6.6407
		STDEV	12.2510	2.3550	3.6243	9.7422
	Improved	Mean	32.2963	8.7818	2.6775	5.6776
		STDEV	29.7758	11.7804	3.4263	10.0101

5.2.2 Real images

Two fundamental matrices with real images shown in Figure 8 are generated as follows:  $F_{RANSAC}$  is computed by the original RANSAC approach;  $F_{pro}$  is estimated using our improved approach.

$$F_{RANSAC} = \begin{pmatrix} -3.6666e-007 & -2.5992e-005 & 8.8458e-003 \\ 2.5965e-005 & 1.0040e-007 & -1.3910e-002 \\ -9.0006e-003 & 1.1450e-002 & 9.9976e-001 \end{pmatrix}$$

$$F_{pro} = \begin{pmatrix} 1.4221e-007 & -2.2705e-005 & 7.7440e-003 \\ 2.2965e-005 & -1.0493e-007 & -1.1206e-002 \\ -8.4282e-003 & 9.0704e-003 & 1.0000e+000 \end{pmatrix}$$

fundamental matrices ( $F_{RANSAC}, F_{pro}$ ) above as well as the time of estimating the fundamental matrices. 89 pairs are generated as the initial points set  $M_0$  and 54 pairs are produced using our improved RANSAC approach as the final set  $M_j(j \geq 0)$ . Though the proposed performs a little bit worse than the original with the initial set  $M_0$  in terms of means and standard deviation, it bounds back with  $M_j$ . Besides, the original RANSAC approach estimates a much better fundamental with  $M_j$  than  $M_0$  does. It is obvious that the means and the standard deviations using the final set  $M_j$  are smaller, which means that  $M_j$  much less outliers than  $M_0$  does and successfully removes a great number of outliers. As removing outliers is much stricter and consumes much more time as well, the time consumed of the proposed increases great.

Table 4 shows the means and standard deviations of the discrepancy between points and epipolar lines with both

TABLE 4 Means and standard deviations of the discrepancy between points and epipolar lines using different algorithms and point matches with the time consumed

Points matched		$M_0$	Time Consumed (s)	$M_j(j \geq 0)$	Time Consumed (s)
		89	—	54	—
Original	Mean	3.6724	0.267263	0.8211	0.212777
	STDEV	11.4096		0.7630	
Improved	Mean	4.0887	2.584081	0.7657	2.299936
	STDEV	12.6091		0.5671	

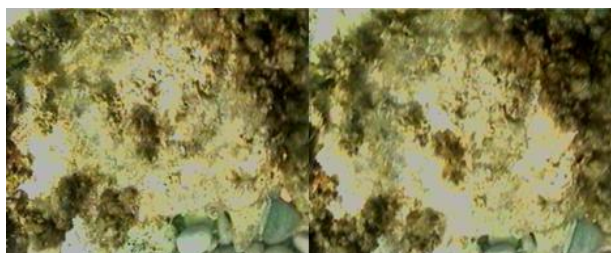


FIGURE 8 Real images used for this experiment

## 6 Conclusions

This paper presents an improved iterative method and a RANSAC approach: the iterative method, utilizing the least-squares technique, extracts several point matches to compute the initial fundamental matrix and weights and determines the computation loop through concerning the Euclidean distance between matched points and epipolar lines. The robust estimation that is a RANSAC approach, adopts a stricter way to remove outliers from the initial matched points. Our approach extends the original RANSAC technique by removing outliers from the points set every five loops after being evaluated to get the optimal points set and then estimates the fundamental matrix through the orthogonal least-square algorithm at each iteration. Meanwhile, different ranges of Gaussian noises are added to experiments with synthetic data to test the robustness of our proposed methods compared to original ones. Besides, in order to evaluate the performances of

these two proposed approaches, the experiments are repeated for 100 times.

Experiments with the improved iterative approach reveal that when both the variance of Gaussian noises and the percentage of outliers go down, performances of the proposed become even better; when the percentage rises with a fixed variance, performances of both approaches decline in an almost identical gradient; when the variance ascends with a constant percentage of outliers, the fundamental matrix estimated by the proposed loses more accuracy, meaning that it is more sensitive to noises.

Though the improved RANSAC method performs a little bit worse than the original with the initial set  $\mathbf{M}_0$  in terms of means and standard deviation, it bounds back with  $\mathbf{M}_j$ . Besides, the means and standard deviations of the original RANSAC approach reduce sharply with the final  $\mathbf{M}_j (j \geq 0)$  that is generated using the improved RANSAC. This reveals that the subset  $\mathbf{M}_j$  consists of much more interior points than  $\mathbf{M}_0$ , inferring that the algorithm of removing outliers used in our approach works successfully and fine.

## Acknowledgments

This work was supported by the Fundamental Research Funds for the Central Universities (Grant No. TD2013-4) and National Natural Science Foundation of China (Grant No. 30901164).

## References

- [1] Luong Q T, Faugeras, O D 1996 The fundamental matrix: theory, algorithms, and stability analysis *Interactional Journal of Computer Vision* **17**(1) 43-75
- [2] Armanque X, Salvi, J 2003 Overall view regarding fundamental matrix estimation *Image and Vision Computing* **21** 205-20
- [3] Zhang Z 1998 Determining the epipolar geometry and its uncertainty: a review *International Journal of Computer Vision* **27**(2) 161-98
- [4] Rousseeuw P, Leroy A 1987 Robust Regression and Outlier Detection *John Wiley&Sons USA*
- [5] Sonka M, Hlavac V, Boyle R 2008 Image Processing, Analysis, and Machine Vision *Thomson Learning USA*
- [6] Hartley R, Zisserman A 2000 Multiple view geometry in computer vision **2** *Cambridge University Press*
- [7] Luong Q T, Faugeras O D 1993 Determining the Fundamental Matrix with Planes Unstability and New Algorithms *Proceedings Conference on Computer Vision and Pattern Recognition New York* 465-84
- [8] Fischler M, Bolles R 1981 Random sample consensus: a paradigm for model fitting with applications to image analysis and automated cartography *Communications of the ACM* **24** 381-5
- [9] Huang J, Lai S H, Cheng C M 2007 Robust Fundamental Matrix Estimation with Accurate Outlier Detection *Journal of Information Science and Engineering* **23**(4) 1213-25
- [10] Harris C, Stephens M 1988 A combined corner and edge detector *Proceedings of the 4th Alvey vision Conference University of Manchester* 147-51
- [11] Zhang Z, Loop C 2001 Estimating the fundamental matrix by transforming image points in projective space *Computer Vision and Image Understanding* **82**(2) 174-80
- [12] Zhang Z, Deriche R, Faugeras O, Luong Q T 1995 A robust technique for matching two uncalibrated images through the recovery of the unknown epipolar geometry *Artificial Intelligence Journal* **78**(5) 87-119

## Authors



Jiangming Kan, 02.01.1976, China.

**Current position, grades:** associate professor in Beijing Forestry University.

**University studies:** PhD degree in forestry engineering from Beijing Forestry University, P.R. China in 2009.

**Scientific interest:** computer vision and intelligent control.

	<p><b>Chuandong Zhan, 21.11.1990, China.</b></p> <p><b>Current position, grades:</b> master student at School of Technology, Beijing Forestry University, China. <b>Scientific interest:</b> forest equipment and automation for forestry engineering.</p>
	<p><b>Shuo Feng, 04.03.1990, China.</b></p> <p><b>University studies:</b> BE degree in Specialty of Electrical Engineering and Automatization at Beijing Forestry University in July 2013, Master degree from Nanyang Technological University in July 2014. <b>Scientific interest:</b> image processing.</p>
	<p><b>Wenbin Li, 23.11.1962, China.</b></p> <p><b>University studies:</b> MS and PhD degrees in Shizuoka University and Ehime University, Japan, in 1987, and 1990, respectively. <b>Scientific interest:</b> forest machinery automation and intelligent.</p>